Received: *December 4, 1998,* Accepted: *February 10, 1999*

D-branes on three-dimensional nonabelian orbifolds

Tomomi Muto

Institute of Physics, University of Tokyo
Komaba, Tokyo 153
E-mail: muto@hep1.c.u-tokyo.ac.jp

Abstract: We study D-branes on a three complex dimensional nonabelian orbifold $\mathbb{C}^3/\Gamma$ with Γ a finite subgroup of $SU(3)$. We present general formulae necessary to obtain quiver diagrams which represent the gauge group and the spectrum of the D-brane worldvolume theory for dihedral-like subgroups $\Delta(3n^2)$ and $\Delta(6n^2)$. It is found that the quiver diagrams have a similar structure to webs of branes.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

JHEP02(1999)008

Contents

1. Introduction

D-branes serve as probes to study short distance structure of spacetime. From a D-brane point of view, spacetime emerges from moduli space of D-brane worldvolume gauge theory and spacetime coordinates are promoted to noncommuting matrices. It is very different from geometric pictures based on general relativity and fundamental strings, so it is natural to ask whether various aspects of spacetime are modified or not if we probe spacetime by D-branes. Investigations in this direction have been developed in recent years. Especially, D-branes on orbifolds have been intensively studied.

D-branes on an orbifold $\mathbb{C}^2/\Gamma$ with Γ a finite subgrop of $SU(2)$ were investigated in [1, 2]. In this case, D-brane worldvolume gauge theory has $\mathcal{N} = 2$ supersymmetry. Finite subgroups of $SU(2)$ are classified into ADE series: A-type subgroups are abelian, while D-type and E-type subgroups are nonabelian. The gauge group and the spectrum of the D-brane gauge theory are represented by a quiver diagram, which corresponds to the Dynkin diagram of ADE affine Lie algebra.

For the $\mathbb{C}^3/\Gamma$ case with Γ a finite subgrop of $SU(3)$, D-brane worldvolume gauge theory has $\mathcal{N} = 1$ supersymmetry. Abelian orbifolds ($\Gamma = \mathbb{Z}_n, \mathbb{Z}_n \times \mathbb{Z}_{n'}$) were investigated in [3, 4, 5, 6]. The phase structure of the Kahler muduli space is investigated and it is shown that only geometric phases appear by using toric methods. It is also

JHEP02(1999)008

shown that topology changing process occurs as in the analyses based on fundamental strings [7, 8]. On the other hand, nonabelian cases have not been studied so far. But it is known that finite subgroups of $SU(3)$ have a similar classification to the ADE classification in the $SU(2)$ case [9]. That is, finite subgroups of $SU(3)$ other than $SU(2)$ and direct products of abelian phase groups fall into 2 series:

- The analogues of dihedral subgroups of $SU(2)$ (D-type);
  $\Delta(3n^2)$ (n is a positive integer)
  $\Delta(6n^2)$ (n is a positive even integer)

- The analogues of exceptional subgroups of $SU(2)$ (E-type);
  $\Sigma(60),\ \Sigma(168),\ \Sigma(360x),\ \Sigma(36x), \Sigma(72x), \Sigma(216x)\quad (x = 1, 3)$

Here the number in braces is the order of the group.

The purpose of the present paper is to investigate D-branes on $\mathbb{C}^3/\Gamma$ with Γ a nonabelian finite subgroup of $SU(3)$, and study what kinds of gauge groups and matter contents are allowed for the D-brane worldvolume gauge thory. However, when we were preparing this manuscript, we received papers [10, 11] which treat similar problems. The complete list of gauge groups and matter contents of D-brane world volume theory for Σ-type subgroups is given in [10], so, in this paper, we concentrate on Δ-type subgroups.

The organization of this paper is as follows. In section 2, we review a prescription to obtain worldvolume gauge theory of a D-brane on $\mathbb{C}^3/\Gamma$. In section 3, we present general formulae which are necessary to obtain the spectrum of the gauge theory of the D-brane on $\mathbb{C}^3/\Gamma$ with $\Gamma = \Delta(3n^2)$ and $\Delta(6n^2)$. It is also pointed out that the formulae have a structure that resembles the condition for a three string junction[12] or a web of (p, q) 5-branes [13]. Section 4 contains discussions. Group theoretical properties of $\Delta(3n^2)$ and $\Delta(6n^2)$ are given in the appendix.

2. Orbifold projection

In this section, we recapitulate the prescription to construct D-brane worldvolume gauge theory on an orbifold $\mathbb{C}^3/\Gamma$. We start with N parallel D3-branes on $\mathbb{C}^3$ where $N = |\Gamma|$ is the order of Γ. The D-brane worldvolume theory is $\mathcal{N} = 4$ supersymmetric 4-dimensional $U(N)$ gauge theory. The bosonic field content are three complex adjoint scalars X^μ ($\mu = 1, 2, 3$) and a $U(N)$ gauge field A. Then we project this theory into Γ invariant space. This condition is expressed as

$$R_{reg} A R_{reg}^{-1} = A\,, \tag{2.1}$$

$$(R_3)_{\mu\nu} R_{reg} X^\nu R_{reg}^{-1} = X^\mu\,, \tag{2.2}$$

where R_{reg} is the $N \times N$ regular representation which acts on Chan-Paton index and R_3 is a 3-dimensional representation which acts on spacetime index μ. R_3 defines

JHEP02(1999)008

how Γ acts on $\mathbb{C}^3$ to form the quotient singularity. The regular representation R_{reg} has the following form

$$R_{reg} = \oplus_{a=1}^{r} N_a R^a \,, \tag{2.3}$$

where R^a is an irreducible representation, $N_a = \dim R^a$ and r is the number of irreducible representations. Due to the condition (2.1), gaauge symmetry of the projected theory becomes

$$\prod_{a=1}^{r} U(N_a) \,. \tag{2.4}$$

The chiral matter obtained after the projection (2.2) can be found by computing the tensor product of the 3-dimensional representation R_3 and an irreducible representation R^a,

$$R_3 \otimes R^a = \oplus_{b=1}^{r} n_{ab}^{\mathbf{3}} R^b \,. \tag{2.5}$$

$n_{ab}^{\mathbf{3}}$ represents the number of fields which transforms as $N_a \otimes \bar{N}_b$ under $U(N_a) \times U(N_b)$.

The gauge group and the spectrum are summarized in a quiver diagram. A quiver diagram consists of r nodes and arrows which connect these nodes: $n_{ab}^{\mathbf{3}}$ represents the number of arrows from the a-th node to the b-th node. So once we calculate the coefficient $n_{ab}^{\mathbf{3}}$, we can obtain the field content.

Now we briefly review how to calculate the coeficient $n_{ab}^{\mathbf{3}}$. A reducible representation R_{red} is decomposed into a direct sum of irreducible representations,

$$R_{red} = \oplus_{a=1}^{r} n_a R^a \,. \tag{2.6}$$

We denote the character for an element $g \in \Gamma$ in a representation R^i as $\chi^i(g)$, then we have the following equation corresponding to the decomposition (2.6),

$$\chi^{red}(g) = \sum_{a=1}^{r} n_a \chi^a(g) \,. \tag{2.7}$$

By using the orthogonality condition for the irreducible representations

$$\frac{1}{|\Gamma|} \sum_{g \in \Gamma} \chi^a(g) \chi^b(g)^* = \delta_{ab} \,, \tag{2.8}$$

we can express the coeficient n_a in the equation (2.6) as

$$n_a = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \chi(g) \chi^a(g)^* \,. \tag{2.9}$$

For a direct product of representations

$$R^i \otimes R^j = R^k \,, \tag{2.10}$$

the following relation holds

$$\chi^i(g) \chi^j(g) = \chi^k(g) \,. \tag{2.11}$$

JHEP02(1999)008

Combining equations (2.9) and (2.11), the coefficient $n^{\mathbf{3}}_{ab}$ in the tensor product (2.5) is expressed as

$$n^{\mathbf{3}}_{ab} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \chi^3(g) \chi^a(g) \chi^b(g)^* \,. \tag{2.12}$$

The elements of Γ are classified into conjugacy classes. Characters are common for all elements in the same conjugacy class, so the expression (2.12) can be rewritten as

$$n^{\mathbf{3}}_{ab} = \frac{1}{|\Gamma|} \sum_{c=1}^{r} |C_c| \chi^3(C_c) \chi^a(C_c) \chi^b(C_c)^* \,. \tag{2.13}$$

where r is the number of conjugacy classes, which is the same as the number of irreducible representations, and $|C_a|$ is the number of elements of the class C_a.

3. D-branes on $\mathbb{C}^3/\Delta(3n^2)$ and $\mathbb{C}^3/\Delta(6n^2)$

In this section, we present formulae for tensor products which represent the spectrum of the D-brane gauge theory on $\mathbb{C}^3/\Delta(3n^2)$ and $\mathbb{C}^3/\Delta(6n^2)$. We discuss physical implications of the formulae. Group theoretical properties of these subgrops are summarized in the appendix.

3.1 $\Delta(3n^2)$ case

We first consider the case with $n \notin 3\mathbb{Z}$. In this case, the irreducible representations consists of three 1-dimensional representations and $(n^2-1)/3$ 3-dimensional representations. So the gauge group of the corresponding worldvolume theory is $U(1)^3 \times U(3)^{(n^2-1)/3}$. We denote the 1-dimensional representations as R_1^α ($\alpha = 0,1,2$), where R_1^0 is the trivial representation. The 3-dimensional representations are labeled by two integers (m_1, m_2) with $(m_1, m_2) \neq (0,0)$. These integers are defined modulo n and furthermore there are equivalence relations among the representations $R_3^{(m_1,m_2)}$,

$$R_3^{(m_1,m_2)} = R_3^{(-m_1+m_2,-m_1)} = R_3^{(-m_2,m_1-m_2)} \,. \tag{3.1}$$

So we can restrict the region of (m_1, m_2) as follows.

$$0 \leq 2m_2 - m_1 < n \,, \qquad -n < m_2 - 2m_1 \leq 0 \tag{3.2}$$

and $(m_1, m_2) \neq (0,0)$. Hence $R_3^{(0,0)}$ is not an irreducible 3-dimensional representation. Instead, we define

$$R_3^{(0,0)} \equiv R_1^1 \oplus R_1^2 \oplus R_1^3 \,. \tag{3.3}$$

Then tensor products of $R_3^{(m_1,m_2)}$ and the irreducible representations are given by a rather compact form,

$$R_3^{(m_1,m_2)} \otimes R_1^\alpha = R_3^{(m_1,m_2)}, \tag{3.4}$$
$$R_3^{(m_1,m_2)} \otimes R_3^{(l_1,l_2)} = R_3^{(l_1+m_1,l_2+m_2)} \oplus R_3^{(l_1-m_2,l_2+m_1-m_2)} \oplus R_3^{(l_1-m_1+m_2,l_2-m_1)} \,. \tag{3.5}$$

JHEP02(1999)008

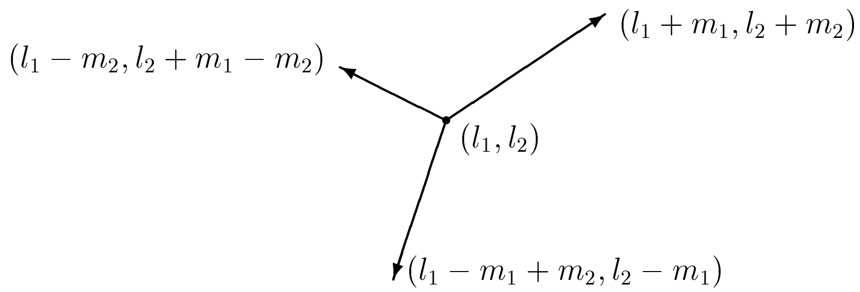

Figure 1: Parts of the quiver diagram for $\mathbb{C}^3/\Delta(3n^2)$.

As noted in the last section, $R_3^{(l_1,l_2)}$ is taken to be an irreducible representation, hence, $(l_1, l_2) \neq (0, 0)$. However we can take l_i to be any integer since the second equation (3.5) with $(l_1, l_2) = (0, 0)$ is consisent with the first one (3.4) under the definition (3.3). Therefore the structure of the tensor products and the corresponding quiver diagram is essentially expressed by the second equation (3.5).

As we noted earlier, the quiver diagram consists of nodes associated with the irreducible representations and arrows associated with the matter contents. In the present case, the irreducible representations are labeled by two integers modulo n, so we can put the nodes on a lattice $\mathbb{Z}_n \times \mathbb{Z}_n$ with some equivalence relations. The equation (3.5) shows that there are three arrows which start from (l_1, l_2). The terminal points are $(l_1 + m_1, l_2 + m_2)$, $(l_1 - m_1 + m_2, l_2 - m_1)$ and $(l_1 - m_2, l_2 - m_1 + m_2)$. This part of the quiver diagram is given in figure 1. The total quiver diagram is obtained by putting such arrows to each nodes in $\mathbb{Z}_n \times \mathbb{Z}_n$ and identifying the nodes according to the equivalence relations (3.1). (To be exact, the node at $(0, 0)$ must be splitted into three nodes due to the equation (3.3).)

Now we comment on the structure of the quiver diagram. The figure 1 graphically resembles a three string junction [12] or a web of (p, q) 5-branes [13]. In fact, relations between junctions of (p, q) strings and representations of algebras were clarified in [14] for the cases of ADE Lie algebras. In [15], it was also pointed out that local geometry of a certain Calabi-Yau threefold has a realization in terms of a web of (p, q) 5-branes. So it is quite interesting to make the connection between quiver diagrams, brane configurations and geometric singularities more definite for the cases considered in this paper.

Here it is worth mentioning to a relation to so-called brane box models [16]. The finite subgroup $\Delta(3n^2)$ (and $\Delta(6n^2)$) can be understood as a generalization of $\mathbb{Z}_n \times \mathbb{Z}_n$. So the orbifolds studied in this paper are generalizations of orbifolds $\mathbb{C}^3/\mathbb{Z}_n \times \mathbb{Z}_n$. For a D-brane on $\mathbb{C}^3/\mathbb{Z}_n \times \mathbb{Z}_n$, we have a counterpart of the equation (3.5),

$$R_3^{(m_1,m_2)} \otimes R_1^{(l_1,l_2)} = R_1^{(l_1+m_1,l_2+m_2)} \oplus R_1^{(l_1-m_1,l_2)} \oplus R_1^{(l_1,l_2-m_2)} .$$

The corresponding part of the quiver diagram is given in figure 2.

JHEP02(1999)008

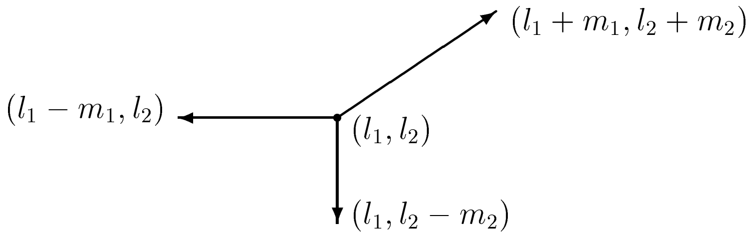

Figure 2: Parts of the quiver diagram for $\mathbb{C}^3/\mathbb{Z}_n \times \mathbb{Z}_n$.

The brane box is a model that gives the same gauge theory as the D-brane worldvolume theory on $\mathbb{C}^3/\mathbb{Z}_n \times \mathbb{Z}_n$. In the brane box model, the figure 2 is directly related to a brane configuration; two types of NS-branes cross orthogonally to each other. So it is natural to expect that the figure 1 represents some brane configuration in which branes cross each other at a certain angle. It would be interesting to study brane configurations from this viewpoint.

Next we consider the case where $n \in 3\mathbb{Z}$. In this case, the irreducible representations consists of nine 1-dimensional representations and $n^2/3 - 1$ 3-dimensional representations. So the gauge group is $U(1)^9 \times U(3)^{n^2/3-1}$. We denote the 1-dimensional representations as R_1^α ($\alpha = 0, 1, \ldots, 8$), where R_1^0 is the trivial representation. The 3-dimensional representations are labeled by two integers (m_1, m_2) where $(m_1, m_2) \neq (0,0), (n/3, 2n/3), (2n/3, n/3)$. As in the $n \notin 3\mathbb{Z}$ case, there are equivalence relations (3.1), so we can restrict the region of (m_1, m_2) as (3.2). By defining

$$R_3^{(0,0)} \equiv R_1^0 \oplus R_1^1 \oplus R_1^2\,, \tag{3.6}$$

$$R_3^{(n/3,2n/3)} \equiv R_1^3 \oplus R_1^4 \oplus R_1^5\,, \tag{3.7}$$

$$R_3^{(2n/3,n/3)} \equiv R_1^6 \oplus R_1^7 \oplus R_1^8\,, \tag{3.8}$$

tensor products of $R_3^{(m_1,m_2)}$ and the irreducible representations are given as follows,

$$R_3^{(m_1,m_2)} \otimes R_1^{0,1,2} = R_3^{(m_1,m_2)}\,, \tag{3.9}$$

$$R_3^{(m_1,m_2)} \otimes R_1^{3,4,5} = R_3^{(n/3+m_1,2n/3+m_2)}\,, \tag{3.10}$$

$$R_3^{(m_1,m_2)} \otimes R_1^{6,7,8} = R_3^{(2n/3+m_1,n/3+m_2)}\,, \tag{3.11}$$

$$R_3^{(m_1,m_2)} \otimes R_3^{(l_1,l_2)} = R_3^{(l_1+m_1,l_2+m_2)} \oplus R_3^{(l_1-m_2,l_2+m_1-m_2)} \oplus R_3^{(l_1-m_1+m_2,l_2-m_1)}. \tag{3.12}$$

Again the first three equations are consistent with the last one, so the structure of the tensor product is essentially expressed by the last equation (3.12). Although the expression is the same as the equation (3.5) in the $n \notin 3\mathbb{Z}$ case, the structure is different. For example, if we take $m_1 + m_2 \in 3\mathbb{Z}$, all the arrows in the quiver diagram have components of the form $(m, -m + 3\mathbb{Z})$ with $m \in \mathbb{Z}$. In this case, not all the nodes are connected in contrast to the $n \notin 3\mathbb{Z}$ case.

JHEP02(1999)008

3.2 $\Delta(6n^2)$ case

We now turn to the subgroup $\Delta(6n^2)$. We first consider the case where $n \notin 3\mathbb{Z}$. In this case, the irreducible representations consists of two 1-dimensional representations, one 2-dimensional representation, $2(n-1)$ 3-dimensional representations and $(n^2-3n+2)/6$ 6-dimensional representations. So the gauge group is $U(1)^2 \times U(2) \times U(3)^{2(n-1)} \times U(6)^{(n^2-3n+2)/6}$. We denote the 1-dimensional representations as R_1^α ($\alpha = 0, 1$), the 2-dimensional representation as R_2 and the 3-dimensional representations as $R_3^{(m,m)t}$, where $m = 1, 2, \ldots, n-1$ and t takes values in $\mathbb{Z}_2$. The 6-dimensional representations are labeled by two integers (m_1, m_2) where $(m_1, m_2) \neq (n, n), (n, 0), (0, n)$. As in the $\Delta(3n^2)$ case, these integers are defined modulo n and there are relations among the representations $R_6^{(m_1,m_2)}$,

$$\begin{aligned} R_6^{(m_1,m_2)} &= R_6^{(-m_1+m_2,-m_1)} = R_6^{(-m_2,m_1-m_2)} = \\ &= R_6^{(m_2,m_1)} = R_6^{(-m_2+m_1,-m_2)} = R_6^{(-m_1,m_2-m_1)} . \end{aligned} \tag{3.13}$$

So we can restrict the region of (m_1, m_2) as follows.

$$2m_2 - m_1 \geq 0 , \qquad m_2 - 2m_1 > -n , \qquad m_1 - m_2 > 0 . \tag{3.14}$$

By defining

$$R_3^{(0,0)t} \equiv \delta_{t0} R_1^0 \oplus \delta_{t1} R_1^1 \oplus R_2 , \tag{3.15}$$

$$R_6^{(m,m)} \equiv R_3^{(m,m)0} \oplus R_3^{(m,m)1} , \tag{3.16}$$

tensor products of R_3 and the irreducible representations are given by a rather compact form,

$$R_3^{(m,m)t} \otimes R_1^\alpha = R_3^{(m,m)t+\alpha} , \tag{3.17}$$

$$R_3^{(m,m)t} \otimes R_2 = R_6^{(m,m)} , \tag{3.18}$$

$$R_3^{(m,m)t} \otimes R_3^{(l,l)t'} = R_3^{(l+m,l+m)t+t'} \oplus R_6^{(l,l-m)} , \tag{3.19}$$

$$R_3^{(m,m)t} \otimes R_6^{(l_1,l_2)} = R_6^{(l_1+m,l_2+m)} \oplus R_6^{(l_1,l_2-m)} \oplus R_6^{(l_1-m,l_2)} . \tag{3.20}$$

As in the $\Delta(3n^2)$ case, the structure of the tensor products (and the associated quiver diagram) is essentially expressed by the last equation (3.20). This equation has a similar structure to the $\Delta(3n^2)$ case except the equivalence relations among the representations.

Next we consider the case where $n \in 3\mathbb{Z}$. In this case, the irreducible representations consists of two 1-dimensional representations, four 2-dimensional representations, $2(n-1)$ 3-dimensional representations and $(n^2-3n)/6$ 6-dimensional representations. So the gauge group is $U(1)^2 \times U(2)^4 \times U(3)^{2(n-1)} \times U(6)^{(n^2-3n)/6}$. We denote the 2-dimensional representations as R_2^α ($\alpha = 0, \ldots, 3$) and use the same

JHEP02(1999)008

notations as the $n \notin 3\mathbb{Z}$ case for other representations. As in the $n \notin 3\mathbb{Z}$ case, there are equivalence relations (3.13), so we can restrict the region of (m_1, m_2) as (3.14). By defining

$$
\begin{aligned}
R_3^{(0,0)t} &\equiv \delta_{t0} R_1^0 \oplus \delta_{t1} R_1^1 \oplus R_2^0 \,, &(3.21)\\
R_6^{(m,m)} &\equiv R_3^{(m,m)0} \oplus R_3^{(m,m)1} \,, &(3.22)\\
R_6^{(2n/3,n/3)} &\equiv R_2^1 \oplus R_2^2 \oplus R_2^3 \,, &(3.23)
\end{aligned}
$$

tensor products of R_3 and the irreducible representations are given as follows,

$$
\begin{aligned}
R_3^{(m,m)t} \otimes R_1^{\alpha} &= R_3^{(m,m)t+\alpha} \,, &(3.24)\\
R_3^{(m,m)t} \otimes R_2^0 &= R_6^{(m,m)} \,, &(3.25)\\
R_3^{(m,m)t} \otimes R_2^{1,2,3} &= R_6^{(2n/3+m,n/3+m)} \,, &(3.26)\\
R_3^{(m,m)t} \otimes R_3^{(l,l)t'} &= R_3^{(l+m,l+m)t+t'} \oplus R_6^{(l,l-m)} \,, &(3.27)\\
R_3^{(m,m)t} \otimes R_6^{(l_1,l_2)} &= R_6^{(l_1+m,l_2+m)} \oplus R_6^{(l_1,l_2-m)} \oplus R_6^{(l_1-m,l_2)} \,, &(3.28)
\end{aligned}
$$

Again, the structure of the tensor product is essentially expressed by the last equation (3.28).

4. Discussion

In this paper, we have considered a D3-brane on an nonabelian orbifold $\mathbb{C}^3/\Gamma$ with $\Gamma = \Delta(3n^2)$ and $\Delta(6n^2)$, finite subgroups of $SU(3)$, which leads to 4-dimentional $\mathcal{N} = 1$ supersymmetric gauge theory. There are many interesting applications.

First, if we take $\Gamma \subset SU(4)$ instead of $SU(3)$, we obtain non-supersymmetric gauge theories. In [17, 18], it is argued that such theories lead to conformal field theories motivated by AdS/CFT correspondence. Much still remains to be done for the non-supersymmetric case.

Secondly, as we discussed in section 3, the models considered in this paper are related to the brane box models or webs of (p, q) 5-branes. Various correspondences between geometric information and brane configurations have been discussed. For example, topology changing process in the geometric picture corresponds to an exchange of branes in the brane configuration picture [19]. On the other hand, topology change is discussed in [11] based on D-branes on nonabelian orbifolds. It is shown that phase structure of Kahler moduli space of nonabelian orbifolds can be partly studied by using toric methods. So it is interesting to investigate whether the D-brane gauge theories considered in this paper are realized by using brane configuration or not, and if possible, compare the topology changing process between the orbifold picture and the brane configuration picture.

JHEP02(1999)008

Finally, we mention to a relation to Mckay correspondence [20]. In the case of $\Gamma \subset SU(2)$, quiver diagrams coincide with Dynkin diagrams of ADE affine Lie algebra, which has a relation to WZW models for $\widehat{SU}(2)$. In [10], it is argued that a similar relation holds for the $\Gamma \subset SU(3)$ case. We hope that the formulae given in this paper are helpful to investigations along this line.

Acknowledgments

This work is supported in part by Japan Society for the Promotion of Science (No. 10-3815).

A. Appendix

In this appendix, we tabulate characters for dihedral-like finite subgroups of $SU(3)$, $\Delta(3n^2)$ and $\Delta(6n^2)$.

A.1 $\Delta(3n^2)$ case

The group $\Delta(3n^2)$ with n a positive integer is given by the following elements,

$$A_{i,j} = \begin{pmatrix} \omega_n^i & 0 & 0 \\ 0 & \omega_n^j & 0 \\ 0 & 0 & \omega_n^{-i-j} \end{pmatrix}, \qquad C_{i,j} = \begin{pmatrix} 0 & 0 & \omega_n^i \\ \omega_n^j & 0 & 0 \\ 0 & \omega_n^{-i-j} & 0 \end{pmatrix}, \qquad E_{i,j} = \begin{pmatrix} 0 & \omega_n^i & 0 \\ 0 & 0 & \omega_n^j \\ \omega_n^{-i-j} & 0 & 0 \end{pmatrix},$$

where $\omega_n = e^{2\pi i/n}$ and $0 \le i, j < n$. In the following expressions, if i (or j) is out of this region, it must be replaced by $i + n\mathbb{Z}$ so that $0 \le i + n\mathbb{Z} < n$.

	C_1	C_2^{ij}	C_3	C_4
$\lvert C_a \rvert$	1	3	n^2	n^2
#classes	1	$(n^2-1)/3$	1	1
R_1^k	1	1	ω_3^k	ω_3^{2k}
$R_3^{(m_1,m_2)}$	3	$\omega_n^{m_1 i + m_2 j} + \omega_n^{-m_1(i+j)+m_2 i} + \omega_n^{m_1 j - m_2(i+j)}$	0	0
$(0 \le k \le 2,\ 0 \le 2m_2 - m_1 < n,\ -n < m_2 - 2m_1 \le 0,\ (m_1, m_2) \ne (0,0))$				

Table 1: Character table for $\Delta(3n^2)$, $(n \notin 3\mathbb{Z})$

$$\begin{aligned} C_1 &= \{A_{0,0}\}, \\ C_2^{ij} &= \{A_{i,j}, A_{-i-j,i}, A_{j,-i-j}\}, \qquad (n < 2i + j < 2n,\ n < i + 2j < 2n) \\ C_3 &= \{C_{i,j} : 0 \le i, j < n\}, \\ C_4 &= \{E_{i,j} : 0 \le i, j < n\}. \end{aligned}$$

JHEP02(1999)008

	C_1^l	C_2^{ij}	C_3^l	C_4^l
$\vert C_a\vert$	1	3	$n^2/3$	$n^2/3$
#classes	3	$n^2/3-1$	3	3
R_1^k	1	1	ω_3^k	ω_3^{2k}
R_1^{k+3}	1	ω_3^{i-j}	ω_3^{k+l}	ω_3^{2k+l}
R_1^{k+6}	1	ω_3^{j-i}	ω_3^{k+2l}	ω_3^{2k+2l}
$R_3^{(m_1,m_2)}$	$3\omega_3^{l(m_1+m_2)}$	$\omega_n^{m_1 i+m_2 j}+\omega_n^{-m_1(i+j)+m_2 i}+\omega_n^{m_1 j-m_2(i+j)}$	0	0
$(0\le k\le 2,\ 0\le 2m_2-m_1<n,\ -n<m_2-2m_1\le 0,\ (m_1,m_2)\neq(0,0))$				

Table 2: Character table for $\Delta(3n^2)$ $\quad(n\in 3\mathbb{Z})$

$$
\begin{aligned}
C_1^l &= \{A_{ln/3,ln/3}\}\,, \qquad (l=0,1,2)\,,\\
C_2^{ij} &= \{A_{i,j}, A_{-i-j,i}, A_{j,-i-j}\}\,, \qquad (n<2i+j<2n,\ n<i+2j<2n)\,,\\
C_3^l &= \{C_{i+3\mathbb{Z}+l,i} : 0\le i<n\}\,, \qquad (l=0,1,2)\,,\\
C_4^l &= \{E_{i+3\mathbb{Z}+l,i} : 0\le i<n\}\,, \qquad (l=0,1,2)\,.
\end{aligned}
$$

A.2 $\Delta(6n^2)$ case

The group $\Delta(6n^2)$ with n a positive even integer is given by $A_{i,j}$, $C_{i,j}$, $E_{i,j}$ and the following matrices.

$$
B_{i,j}=\begin{pmatrix}\omega_n^i & 0 & 0\\ 0 & 0 & \omega_n^j\\ 0 & \omega_n^{n/2-i-j} & 0\end{pmatrix},\quad
D_{i,j}=\begin{pmatrix}0 & \omega_n^i & 0\\ \omega_n^j & 0 & 0\\ 0 & 0 & \omega_n^{n/2-i-j}\end{pmatrix},\quad
F_{i,j}=\begin{pmatrix}0 & 0 & \omega_n^i\\ 0 & \omega_n^j & 0\\ \omega_n^{n/2-i-j} & 0 & 0\end{pmatrix}.
$$

	C_1	C_2^i	C_3^{ij}	C_4	C_5^l
$\vert C_a\vert$	1	3	6	$2n^2$	$3n$
#classes	1	$n-1$	$(n^2-3n+2)/6$	1	n
R_1^t	1	1	1	1	$(-1)^t$
R_2	2	2	2	-1	0
$R_3^{(m,m)t}$	3	χ_3^{ii}	χ_3^{ij}	0	$(-1)^t\omega_n^{ml}$
$R_6^{(m_1,m_2)}$	6	χ_6^{ii}	χ_6^{ij}	0	0
$(0<m<n,\ t\in\mathbb{Z}_2,\ 2m_2-m_1\ge 0,\ m_2-2m_1>-n,\ m_1-m_2>0)$					

Table 3: Character table for $\Delta(6n^2)$ $\quad(n\notin 3\mathbb{Z})$

$$
\begin{aligned}
\chi_3^{ij} &= \omega_n^{m(i+j)}+\omega_n^{-mi}+\omega_n^{-mj}\\
\chi_6^{ij} &= \omega_n^{m_1 i+m_2 j}+\omega_n^{-m_1(i+j)+m_2 i}+\omega_n^{m_1 j-m_2(i+j)}\\
&\quad+\omega_n^{m_1 j+m_2 i}+\omega_n^{-m_1(i+j)+m_2 j}+\omega_n^{m_1 i-m_2(i+j)}
\end{aligned}
$$

JHEP02(1999)008

$$
\begin{aligned}
C_1 &= \{A_{0,0}\}\,, \\
C_2^i &= \{A_{i,i}, A_{-2i,i}, A_{i,-2i}\}\,, \quad (0 < i < n)\,, \\
C_3^{ij} &= \{A_{i,j}, A_{-i-j,i}, A_{j,-i-j}, A_{j,i}, A_{-i-j,j}, A_{i,-i-j}\},\ (i+2j > n, 2i+j < 2n, i-j > 0), \\
C_4 &= \{C_{i,j}, E_{i,j} : 0 \le i,j < n\}\,, \\
C_5^l &= \{B_{i,l-i}, D_{i,n/2-l}, F_{n/2-l,j} : 0 \le i,j < n\}\,, \quad (0 \le l < n)\,.
\end{aligned}
$$

	C_1^l	C_2^i	C_3^{ij}	C_4^l	C_5^l
$\lvert C_a\rvert$	1	3	6	$2n^2/3$	$3n$
#classes	3	$n-3$	$(n^2-3n+6)/6$	3	n
R_1^t	1	1	1	1	$(-1)^t$
R_2^0	2	2	2	-1	0
R_2^k	2	2	$\omega_3^{i-j}+\omega_3^{j-i}$	$\omega_3^{k+l}+\omega_3^{-k-l}$	0
$R_3^{(m,m)t}$	$3\omega_3^{2lm}$	χ_3^{ii}	χ_3^{ij}	0	$(-1)^t\omega_n^{ml}$
$R_6^{(m_1,m_2)}$	$6\omega_3^{l(m_1+m_2)}$	χ_6^{ii}	χ_6^{ij}	0	0
$(0 \le k \le 2,\ 0 < m < n,\ t \in \mathbb{Z}_2,\ 2m_2 - m_1 \ge 0,\ m_2 - 2m_1 > -n,\ m_1 - m_2 > 0)$					

Table 4: Character table for $\Delta(6n^2)$ $\quad (n \in 3\mathbb{Z})$

$$
\begin{aligned}
C_1^l &= \{A_{ln/3,ln/3}\}\,, \quad (l = 0,1,2)\,, \\
C_2^i &= \{A_{i,i}, A_{-2i,i}, A_{i,-2i}\}\,, \quad (0 < i < n,\ i \ne n/3, 2n/3)\,, \\
C_3^{ij} &= \{A_{i,j}, A_{-i-j,i}, A_{j,-i-j}, A_{j,i}, A_{-i-j,j}, A_{i,-i-j}\},\ (i+2j > n, 2i+j < 2n, i-j > 0), \\
C_4^l &= \{C_{i,i+3\mathbb{Z}+l}, E_{i+3\mathbb{Z}+l,i} : 0 \le i < n\}\,, \quad (l = 0,1,2)\,, \\
C_5^l &= \{B_{i,l-i}, D_{i,n/2-l}, F_{n/2-l,j} : 0 \le i,j < n\}\,, \quad (0 \le l < n)\,.
\end{aligned}
$$

References

[1] M.R. Douglas and G. Moore, *D-branes, Quivers and ALE Instantons*, hep-th/9603167.

[2] C.V. Johnson and R.C. Myers, *Aspects of Type IIB Theory on ALE Spaces*, *Phys. Rev.* **D 55** (1997) 6382 [hep-th/9610140].

[3] M.R. Douglas, B.R. Greene and D.R. Morrison, *Orbifold Resolution by D-branes*, *Nucl. Phys.* **B 506** (1997) 84 [hep-th/9704151].

[4] T. Muto, *D-branes on Orbifolds and Topology Change*, *Nucl. Phys.* **B 521** (1998) 183 [hep-th/9711090].

[5] B.R. Douglas, *D-brane Topology Changing Transitions*, *Nucl. Phys.* **B 525** (1998) 284 [hep-th/9711124].

JHEP02(1999)008

[6] S. Mukhopadhyay and K. Ray, *Conifolds From D-branes*, *Phys. Lett.* **B 423** (1998) 247 [hep-th/9711131].

[7] P.S. Aspinwall, B.R. Greene and D.R. Morrison, *Calabi-Yau Moduli Space, Mirror Manifolds and Spacetime Topology Change in String Theory*, *Nucl. Phys.* **B 416** (1994) 414 [hep-th/9309097].

[8] E. Witten, *Phases of N=2 Theories In Two Dimensions*, *Nucl. Phys.* **B 403** (1993) 159 [hep-th/9301042].

[9] W.M. Fairbairn, T. Fulton and W.H. Klink, *Finite and Disconnected Subgroups of SU_3 and their Applications to The Elementary Particle Spectrum*, *J. Math. Phys.* **5(8)** (1964) 1038.

[10] A. Hanany and Y.H. He, *Non-Abelian Finite Gauge Theories*, hep-th/9811183.

[11] B.R. Greene, C.I. Lazaroiu and M. Raugas, *D-branes on Nonabelian Threefold Quotient Singularities*, hep-th/9811201.

[12] J.H. Schwarz, *Lectures on Superstring and M theory Dualities*, *Nucl. Phys.* **55B** *(Proc. Suppl.)* (1997) 1 [hep-th/9607201].

[13] O. Aharony, A. Hanany and B. Kol, *Webs of (p,q) 5-branes, Five Dimensional Field Theories and Grid Diagrams*, *J. High Energy Phys.* **01** (1998) 002 [hep-th/9710116].

[14] O. DeWolfe and B. Zwiebach, *String Junctions for Arbitrary Lie Algebra Representations*, *Nucl. Phys.* **B 541** (1999) 509 [hep-th/9804210].

[15] N.C. Leung and C. Vafa, *Branes and Toric Geometry*, *Adv. Theor. Math. Phys.* **2** (1998) 91 [hep-th/9711013].

[16] A. Hanany and A. Uranga, *Brane Boxes and Branes on Singularities*, *J. High Energy Phys.* **05** (1998) 013 [hep-th/9805139].

[17] S. Kachru and E. Silverstein, *4d conformal field theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[18] A. Lawrence, N. Nekrasov and C. Vafa, *On Conformal Theories in Four Dimensions*, *Nucl. Phys.* **B 533** (1998) 199 [hep-th/9803015].

[19] A. Uranga, *Brane Configurations for Branes at Conifolds*, hep-th/9811004.

[20] J. Mckay, *Graphs, Singularities and Finite Groups*, *Proc. Symp. Pure Math.* **37** (1980) 183.

JHEP02(1999)008